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INVESTIGATION OF THE INITIAL STAGE OF SEPARATION FLOW AROUND A CIRCULAR CYLINDER

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The model of potential flow of an ideal incompressible fluid is used extensively in theoretical investigation of nonstationary separation flow around bodies (see [1, 2], for instance). However, within the framework of this model a number of important questions have not yet been solved. Among them is the construction of the asymptotics for the solution in the neighborhood of the initial time. This paper is devoted to an investigation of this question for plane separation flow around a circular cylinder that starts to move from a state of rest.

Let us consider the plane fluid flow around a circular cylinder that occurs as it moves from a state of rest. We assume that the flow occurs with stream separation, which we model by one vortex wake converging with the cylinder outline. We consider the fluid ideal and incompressible, and the flow outside the cylinder and the vortex wake potential.

Let us formulate the problem of determining the kinematic flow parameters in a small neighborhood of the initial time $t = 0$ for certain constraints on the cylinder motion and the vortex wake parameters. Let us introduce a rectangular $O_1x_1y_1$ coordinate system at whose infinite point the fluid is at rest. Let a cylinder L_0 of radius R move at a velocity $-U(t)$ along the O_1x_1 axis (see Fig. 1).

We shall assume that the curvature of the vortex wake contour L_1 is continuous in the direction from A to B_1 while the intensity of the vortex wake $\gamma_1(\tau_1, t)$ has a derivative with respect to τ on L_1 that belongs to the class H^* in the neighborhood of the end B_1 and to the class H on the remaining part [3] (τ is the complex coordinate of a point of the contour L_1 in the complex plane $z_1 = x_1 + iy_1$, and t is the time). We also assume that the fluid velocities are finite everywhere. Consequently, the vortex wake will converge with the streamlined contour along the tangent [4], and its intensity at the point B_1 will be zero.

At a fixed time t in the complex plane z_1 a boundary value problem can be formulated for the complex velocity $\bar{v}(z_1, t)$ analogously to how it is done in [5, 6], on the construction of an analytic function $\bar{v}(z_1, t)$ outside the contours L_0 and L_1 which would satisfy the condition of nonpenetration on L_0, have a given jump on L_1, disappear at infinity, be finite everywhere and satisfy the Thomson theorem on constancy of the circulation of velocity over a closed fluid contour. This problem is a Reimann-Hilbert problem and allows of a unique solution that can be written in the form

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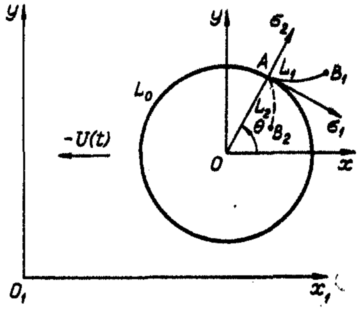

Fig. 1

$$\bar{v}(z_1, t) = -\frac{U^2R^2}{(z_1 - z_0)^2} + \frac{1}{2\pi i}\sum_{k=1,2}\int\limits_{L_k}\frac{\gamma_k(\tau_k)\,d\tau_k}{\tau_k - z_1}, \tag{1}$$

where L_2 is the contour obtained from L_1 by an inversion with respect to the circle L_0, $\gamma_2(\tau_2) = -\gamma_1(\tau_1)\partial\tau_1/\partial\tau_2$; z_0 is the complex coordinates of the point 0.

Therefore, at each time the velocity field is determined by giving the velocity U(t), the contour L_1 of the vortex wake, and the distribution of the vortex intensity there $\gamma_1(\tau_1)$.

By using the Cauchy—Lagrange integral and the condition of no pressure jump on the contour L_1, it can be obtained that the circulation Γ of the vortex wake measured from its end B_1 is conserved at points moving at the velocities $v_0 = (v^+ + v^-)/2$. It hence follows that if the contour L_1 is given in the parametric form $\tau_1 = \tau_1(\Gamma, t)$, then its motion will be described by the equation [5, 6].

$$\frac{\partial\bar{\tau}_1}{\partial t}(\Gamma, t) = \bar{v}_0(\tau_1(\Gamma, t), t). \tag{2}$$

Let us introduce the scalar function of time

$$\gamma_*(t) = \gamma_1(\tau_1^*, t)\left(\frac{\partial\tau_1}{\partial s_1}\right)^*,$$

where the asterisk denotes quantities referring to the separation point, and s_1 is the arc coordinate of a point on the curve L_1 measured from A. The function $\gamma_*(t)$ satisfies the equation [4]

$$\frac{d\gamma_*}{dt} = \gamma_*\left(\frac{\partial v_n^+}{\partial n}\right)^*; \tag{3}$$

n is the external normal to the streamlined contour L_0.

It can be shown that under the assumptions of finiteness of the velocity field and continuity of the curvature of the contour L_1 its asymptotics in the neighborhood of the flow separation point have the form [5, 7].

$$\sigma_2 = -\frac{\sigma_1^2}{2R} + \lambda(t)\sigma_1^{5/2} + o(\sigma_1^{5/2}), \tag{4}$$

where σ_1, σ_2 are the abscissa and ordinate of a point on L_1 in a rectangular Cartesian coordinate system with origin at the point A (see Fig. 1). The axis σ_1 is here directed along the tangent to L_0 while the axis σ_2 is along the external normal, and $\lambda(t)$ is a certain coefficient dependent on the time.

Let us turn to the direct solution of the problem of seeking the asymptotic of the flow in the neighborhood of the initial time. The relationships (1)-(4) allow reduction of this problem to determination of the time dependence of the functions $\gamma_*(t)$, $\lambda(t)$, and $l(t)$, the lengths of the wake projections on the axis σ_1.

Let us introduce the function V = U + v and the complex variable $z = z_1 - z_0$. We let τ_1, τ_2 denote the complex coordinates of points on L_1 and L_2 in the plane z.

We obtain new relationships containing the desired quantities γ_*, λ, l. We convert the expression (3) by calculating $(\partial \bar{V}^+/\partial n)^*$ therein in terms of the limit value $\partial \bar{V}^+/\partial z$. In conformity with the behavior of an integral of Cauchy type near the end of the contour of integration [3] and the Leibnitz formula [8], we obtain from (1)

$$a_0^2 \frac{\partial \bar{V}}{\partial z}(z) = \frac{a_0^2}{2\pi i}\left\{\frac{4\pi i U R^2}{z^3} + \sum_{k=1,2}\left(\frac{\gamma_k(a)}{a-z} + \gamma_k'(a)\ln\frac{b_k - z}{a-z} + \int_{L_k}\frac{\gamma_k'(\tau_k) - \gamma_k'(a)}{\tau_k - z}\,d\tau_k\right)\right\}. \quad (5)$$

The contour integrals in this expression are continuous functions of the variable z in the neighborhood of the point a. The remaining components, except the first, have a singularity at this point. Here $a_0 = a/R$. Substituting here

$$\gamma_2(a) = -\gamma_1(a), \quad \gamma_2'(a) = -\gamma_1'(a) - 2\gamma_1(a)(1/R - k_2)\bar{a}/R,$$

we arrive at the deduction that the coefficient of the singularity $(a - z)^{-1}$ is zero, while for $\ln(a - z)$

$$\frac{a_0^2}{\pi i}\gamma_1(a)\frac{\bar{a}}{R}\left(\frac{1}{R} - k_2\right) = \frac{1}{\pi}\frac{\partial \Gamma}{\partial s_1}\left(\frac{1}{R} - k_2\right).$$

Because of the assumption about the finiteness of the velocity field in the whole flow plane, the condition

$$k_2 = 1/R \quad (6)$$

must be satisfied, which will assure that the coefficient of $\ln(a - z)$ will be zero. Condition (6) denotes agreement of the vortex wake curvature with the curvature of the streamlined contour at the shedding point.

Taking account of the relations (5) and (6), the equality (3) can be reduced after a number of manipulations to the form

$$\gamma_*' = \gamma_* \operatorname{Re}\frac{a_0^2}{2\pi i}\left\{\frac{4\pi i U R^2}{a^3} - \int_{L_1}\left(\frac{1}{(\tau_1 - a)^2} - \frac{1}{(\tau_2 - a)^2}\right)d\Gamma\right\}. \quad (7)$$

Let us obtain still another relationship by using (1) and (2). To do this we calculate the velocity $\bar{V}_0 = (\bar{V}^+ + \bar{V}^-)/2$ at a certain point $\tau \in L_1$ by setting $\tau \neq a$ [3]. We define $\bar{V}_0(a)$ as the limit of $V_0(\tau)$ as $\tau \to a$. The obtained limit can be converted to the form

$$\bar{V}_0(a) = U(1 - R^2/a^2) - \gamma_1(a)/2 + (2\pi i)^{-1}\int_{L_1}(1/(\tau_1 - a) - 1/(\tau_2 - a))\gamma_1(\tau_1)\,d\tau_1.$$

On the other hand $\bar{V}_0(a) = \bar{V}^-(a) + \gamma_1(a)/2$, where $\bar{V}^-(a)$ is the fluid particle velocity under the wake. Equating the right sides of these equalities, we arrive at the desired relationship

$$\bar{V}^-(a) + \gamma_1(a) = U(1 - R^2/a^2) - (2\pi i)^{-1}\int_{L_1}(1/(\tau_1 - a) - 1/(\tau_2 - a))\,d\Gamma. \quad (8)$$

Expressing τ_1, τ_2 in (7) and (8) in terms of σ_1, σ_2, and going over to scalars, we obtain

$$-R\theta' + \gamma_* = 2U\sin\theta + \frac{1}{2\pi}\int_{L_1}\frac{2\sigma_2 + R^{-1}(\sigma_1^2 + \sigma_2^2)}{\sigma_1^2 + \sigma_2^2}\,d\Gamma, \quad (9)$$

$$\gamma_*' = 2\gamma_* U R^{-1}\cos\theta - \frac{\gamma_*}{\pi}\int_{L_1}\frac{2\sigma_1\sigma_2 + R^{-1}\sigma_1(\sigma_1^2 + \sigma_2^2)}{(\sigma_1^2 + \sigma_2^2)^2}\,d\Gamma,$$

where θ is the angular coordinate of the separation point measured counterclockwise from the Ox axis.

Let us investigate the system of relations (9). We note that if the integral components in both equations are discarded, then the system of ordinary differential equations obtained in this manner for the functions $\gamma_*(t)$ and $\theta(t)$ has no solutions satisfying the condition $\gamma_*(0) = 0$. This means that these components should play a substantial role as $t \to 0$. Moreover, the integral term in the second equation should be a quantity of the same order in time and of the same sign as γ'_*/γ_*. It can be hence shown that the integral term in the first equation does not exceed γ_* in order of magnitude and has opposite sign. Setting the acceleration of the point of separation equal to zero at the initial time and $U(t) = U_0 t^\alpha$, $0 < \alpha \leqslant 1$, we conclude that $R\theta'$ is small compared with $2U\sin\theta$. This denotes the power-law form of the time dependence with exponent α of the principal part of $\gamma_*(t)$:

$$\gamma_*(t) = \gamma_0 t^\alpha + o(t^\alpha), \quad 0 < \alpha \leqslant 1. \tag{10}$$

It follows also from (9) that the wake configurations for which $\sigma_2 \leqslant 0$ for all points of the contour are not realized. The quantity $2UR^{-1}\gamma_* \cos\theta$ is an infinitesimal as $t \to 0$ as compared with γ'_*; hence it can be neglected for subsequent calculations.

According to the representation (10), for small times the formulas

$$l - \sigma_1 = \frac{\gamma_0}{2(\alpha+1)} t_1^{\alpha+1}; \tag{11}$$

$$\gamma(\sigma_1) = \gamma_1 (l - \sigma_1)^\omega \tag{12}$$

will hold to higher order accuracy, where t_1 is the time of shedding of a point of the vortex wake having the coordinate σ_1 at the time t from the cylinder:

$$\gamma_1 = \gamma_0^{1/(\alpha+1)} \left(2/(\alpha+1)^{\alpha/(\alpha+1)}\right), \quad \omega = \alpha/(\alpha+1), \quad \gamma(\sigma_1) = -d\Gamma/d\sigma_1. \tag{13}$$

It should be noted that because of (12), the continuity of the curvature of the contour L_1, and its tangency to the contour L_0 at the point A, the function $\gamma_1'(\tau)$ on the contour L_1 will belong to the class H^* in the neighborhood of the end B_1 and satisfy the Hölder condition on its remaining part. This means that (10) does not contradict the assumptions under which the relations (9) were obtained. Moreover, (11) taken for $t_1 = t$ yields the time dependence of the wake length, which diminishes the number of desired functions and permits consideration of a certain function $\lambda_1(l)$ instead of $\lambda(t)$.

Let us express σ_2 in terms of σ_1 by using the expansion (4), while retaining the two first terms, and let us substitute it into (9). We note that if only the first were taken instead of the two, then the system (9) would again not have a solution satisfying the condition $\gamma_*(0) = 0$. For the left and right sides of the second of the relations obtained in this manner to be quantities of the same order, satisfaction of the following equality is necessary

$$\lambda_1(l) = \lambda_0 l^{-3/2} + o(l^{-3/2}).$$

Taking this fact into account, as well as (11)-(13), it can be shown that the relationships (9) are representable in the form

$$\gamma_* = 2U\sin\theta - \frac{1}{\pi}\int_0^l \frac{\lambda_0 l^{-3/2}\sigma_1^{5/2}}{\sigma_1^2 + \lambda_0^2 l^{-3}\sigma_1^5}\gamma(\sigma_1)\,d\sigma_1 + o(l^\omega),$$

$$\gamma'_* = \frac{\gamma_*}{\pi}\int_0^l \frac{2\lambda_0 l^{-3/2}\sigma_1^{7/2}}{(\sigma_1^2 + \lambda_0^2 l^{-3}\sigma_1^5)^2}\gamma(\sigma_1)\,d\sigma_1 + o(l^{2\omega-1}).$$

The dependence of the integrals on l in these relationships can be extracted in the form of multipliers by using (12) by interchanging the variables of integration. Then discarding the remainder terms, we finally obtain

$$\gamma_* = 2U\sin\theta - \lambda_0\pi^{-1}\gamma_1 l^\omega \int_0^1 \frac{\sigma_1^{1/2}}{1+\lambda_0^2\sigma_1^3}(1-\sigma_1)^\omega d\sigma_1, \tag{14}$$

$$\gamma'_* = 2\lambda_0\pi^{-1}\gamma_*\gamma_1 l^{\omega-1}\int_0^1 \frac{\sigma_1^{-1/2}}{(1+\lambda_0^2\sigma_1^3)^2}(1-\sigma_1)^\omega d\sigma_1.$$

The system (14) sets up the connection between the parameters of the vortex wake and the velocity of the cylinder at the time t under consideration. For arbitrary positive U_0 and $0 < \alpha \leqslant 1$ this system permits the unique determination of the quantities γ_0, λ_0, and therefore, the desired functions γ_*, λ, l, which determine the asymptotics of the initial stage of separation flow around a cylinder, because of (1), (4), (11)-(13).

In particular, the solution of (14) for motion with finite acceleration ($\alpha = 1$) is determined by the relationships

$$\frac{\pi}{8\lambda_0} = \int_0^1 \frac{\sigma_1^{-1/2}}{(1+\lambda_0^2\sigma_1^3)^2}(1-\sigma_1)^{1/2}d\sigma_1,$$

$$\gamma_0 = 2U_0\sin\theta\left(1+\lambda_0\pi^{-1}\int_0^1 \frac{\sigma_1^{1/2}}{1+\lambda_0^2\sigma_1^3}(1-\sigma_1)^{1/2}d\sigma_1\right)^{-1}.$$

By using the first, the constant λ_0 can be found numerically to any degree of accuracy. A computation showed that $\lambda_0 = 0.252$. Hence, $\gamma_0 = 1.94\,U_0\sin\theta$ follows.

Therefore, the desired functions will be in this case

$$\gamma_*(t) = \gamma_0 t, \quad \lambda(t) = 8\lambda_0\gamma_0^{-3/2}t^{-3}, \quad l(t) = \gamma_0 t^2/4. \tag{15}$$

The asymptotics obtained can be used for a qualitative analysis and a numerical computation of the intitial stage of the flow around a circular cylinder. For instance, the relationships (15) permit making the deduction that the center of gravity of the vorticity of a vortex wake and its end B_1 in the neighborhood of the time $t = 0$ move along lines making angles of about 6 and 14° respectively with the axis σ_1.

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